Bulg. J. Phys. **44** (2017) 15–30

Quintessential Inflation, Unified Dark Energy and Dark Matter, and Higgs Mechanism*

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Received *1 November 2016*

Abstract. We describe a new type of gravity-matter models where gravity couples in a non-conventional way to two distinct scalar fields providing a unified Lagrangian action principle description of: (a) the evolution of both "early" and "late" Universe – by the "inflaton" scalar field; (b) dark energy and dark matter as a unified manifestation of a single material entity – the "darkon" scalar field. The essential non-standard feature of our models is employing the formalism of non-Riemannian space-time volume forms – alternative generally covariant integration measure densities (volume elements) defined in terms of auxiliary antisymmetric tensor gauge fields. Although being (almost) pure-gauge degrees of freedom, the non-Riemannian space-time volume forms trigger a series of important features unavailable in ordinary gravity-matter models. When including in addition interactions with the electro-weak model bosonic sector we obtain a gravity-assisted generation of electro-weak spontaneous gauge symmetry breaking in the post-inflationary "late" Universe, while the Higgs-like scalar remains massless in the "early" Universe.

PACS codes: 04.50.Kd, 98.80.Jk, 95.36.+x, 95.35.+d, 11.30.Qc,

1 Introduction

Dark energy and dark matter, occupying around 70% and 25% of the matter content of the Universe, respectively, continue to be the two most unexplained "mysteries" in cosmology and astrophysics (for a background, see [1, 2]). In most loose terms dark energy is responsible for the accelerated expansion of today's Universe, i.e., dark energy acts effectively as repulsion force among the galaxies – a phenomenon completely counterintuitive w.r.t. the naive notion about gravity as an attractive force. And vice versa, dark matter holds together

*This article is based on a talk given at the 3rd National Congress on Physical Sciences, 29 Sep. – 2 Oct. 2016, Sofia.

the matter objects inside the galaxies. The adjective "dark" is due to the fact that both these fundamental matter components of the Universe interact only gravitationally, and they do not directly interact with ordinary (baryonic) matter, in particular, they do not interact electromagnetically and thus they remain "dark".

There exist a multitude of proposals for an adequate description of dark energy's and dark matter's dynamics within the framework of standard general relativity or its modern extensions [3–5]. Here we will briefly describe and further extend the basic features of our approach to the above topic [6] (for some earlier works, see also [7]).

Using the method of non-Riemannian spacetime volume-forms (metric-independent generally-covariant integration measure densities or volume elements) [8] we start by constructing from first principles (via Lagrangian action) a new non-canonical cosmological model of gravity interacting with a single scalar field (here called "darkon"), which explicitly yields a self-consistent unified description of dark energy as a dynamically generated cosmological constant, and dark matter as a dust fluid flowing along spacetime geodesics, by unifying them as an exact sum of two separate contributions to the pertinent scalar field energy-momentum tensor. In other words, this unified description shows that dark energy and dark matter may be viewed as two different manifestations of one single matter source – the scalar "darkon" field [6].

Next, extending our formalism of non-Riemannian spacetime volume-forms, we couple the above non-canonical gravity-matter system to a second scalar field – the "inflaton" – in such a way that the "inflaton" dynamics provides a unified description of the evolution of both "early" and "late" Universe [9] – this is a model of "quintessential inflation" [10]. Furthermore, we add interaction with the $SU(2) \times U(1)$ scalar and gauge fields of the electro-weak bosonic sector.

We exhibit in some detail the interplay between the "inflaton" and the "darkon" in the "early" (inflationary) and the "late" (dark energy dominated) epochs of the Universe. Among the principal interesting features is the gravity-assisted generation in the "late" Universe of Higgs-like spontaneous gauge symmetry breaking effective potential for the $SU(2) \times U(1)$ scalar iso-doublet[1]. In the "early" Universe the Higgs-like field remains massless.

2 Hidden Noether Symmetry and Unification of Dark Energy and Dark Matter

First we will consider, following [6], a simple particular case of a non-conventional gravity-scalar-field action – a member of the general class of the "modified-measure" gravity-matter theories [8] (for simplicity we use units with

[1] For a related approach, see [11] based on an old idea by Bekenstein [12].

the Newton constant $G_N = 1/16\pi$)

$$S = \int d^4x\sqrt{-g}\,R + \int d^4x\bigl(\sqrt{-g} + \Phi(C)\bigr)L(u,Y)\,. \tag{1}$$

Here R denotes the standard Riemannian scalar curvature for the pertinent Riemannian metric $g_{\mu\nu}$. In the second term in (1) – the scalar field Lagrangian is coupled *symmetrically* to two mutually independent spacetime volume-forms (integration measure densities or volume elements) – the standard Riemannian $\sqrt{-g} = \sqrt{-\det\|g_{\mu\nu}\|}$ and to an alternative non-Riemannian one

$$\Phi(C) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu C_{\nu\kappa\lambda}\,, \tag{2}$$

where $C_{\mu\nu\lambda}$ is an auxiliary rank 3 antisymmetric tensor field.

$L(u,Y)$ is general-coordinate invariant Lagrangian of a single scalar field $u(x)$

$$L(u,Y) = Y - V(u)\,, \quad Y \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu u\partial_\nu u\,. \tag{3}$$

As a result of the equations of motion w.r.t. "measure" gauge field $C_{\mu\nu\lambda}$ we obtain the following crucial new property of the model (1) – *dynamical constraint* on $L(u,Y)$ alongside with the second-order differential equations of motion w.r.t. u (which now contains the non-Riemannian volume element $\Phi(C)$ (2))

$$\partial_\mu L(u,Y) = 0 \;\longrightarrow\; L(u,Y) = -2M_0 = \text{const}\,,\ \text{i.e.}\ Y = V(u) - 2M_0\,, \tag{4}$$

where M_0 is arbitrary integration constant. The factor 2 in front of M_0 is for later convenience in view of its interpretation as a *dynamically generated cosmological constant*.

Indeed, taking into account (4) the energy-momentum tensor becomes

$$T_{\mu\nu} = -2g_{\mu\nu}M_0 + \Bigl(1 + \frac{\Phi(C)}{\sqrt{-g}}\Bigr)\partial_\mu u\,\partial_\nu u\,, \quad \nabla^\nu T_{\mu\nu} = 0\,. \tag{5}$$

A second crucial property of the model (1) is the existence of a *hidden strongly nonlinear Noether symmetry* due to the presence of the non-Riemannian volume element $\Phi(C)$

$$\delta_\epsilon u = \epsilon\sqrt{Y}\,, \quad \delta_\epsilon g_{\mu\nu} = 0\,, \quad \delta_\epsilon \mathcal{C}^\mu = -\epsilon\frac{1}{2\sqrt{Y}}g^{\mu\nu}\partial_\nu u\bigl(\Phi(C) + \sqrt{-g}\bigr)\,, \tag{6}$$

where $\mathcal{C}^\mu \equiv \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}C_{\nu\kappa\lambda}$. Under (6) the action (1) transforms as: $\delta_\epsilon S = \int d^4x\,\partial_\mu\bigl(L(u,Y)\delta_\epsilon\mathcal{C}^\mu\bigr)$. Then, standard Noether procedure yields a conserved current

$$\nabla_\mu J^\mu = 0\,, \quad J^\mu \equiv -\Bigl(1 + \frac{\Phi(C)}{\sqrt{-g}}\Bigr)\sqrt{2Y}g^{\mu\nu}\partial_\nu u\,. \tag{7}$$

$T_{\mu\nu}$ (5) and J^μ (7) can be cast into a relativistic hydrodynamical form (taking into account (4))

$$T_{\mu\nu} = -2M_0 g_{\mu\nu} + \rho_0 u_\mu u_\nu \,, \quad J^\mu = \rho_0 u^\mu \,, \tag{8}$$

where

$$\rho_0 \equiv \Bigl(1 + \frac{\Phi(C)}{\sqrt{-g}}\Bigr)\, 2Y \,, \quad u_\mu \equiv -\frac{\partial_\mu u}{\sqrt{2Y}} \,, \quad u^\mu u_\mu = -1 \,. \tag{9}$$

For the pressure p and energy density ρ we have accordingly (with ρ_0 as in (9))

$$p = -2M_0 = \mathrm{const} \,, \quad \rho = \rho_0 - p = 2M_0 + \Bigl(1 + \frac{\Phi(C)}{\sqrt{-g}}\Bigr) 2Y \,. \tag{10}$$

Because of the constant pressure ($p = -2M_0$) $\nabla^\nu T_{\mu\nu} = 0$ implies *both* hidden Noether symmetry current $J^\mu = \rho_0 u^\mu$ conservation, as well as *geodesic fluid motion*

$$\nabla_\mu \bigl(\rho_0 u^\mu\bigr) = 0 \,, \quad u_\nu \nabla^\nu u_\mu = 0 \,. \tag{11}$$

Therefore, $T_{\mu\nu} = -2M_0 g_{\mu\nu} + \rho_0 u_\mu u_\nu$ represents an exact sum of two contributions of the two dark species

$$p = p_{\mathrm{DE}} + p_{\mathrm{DM}} \,, \quad \rho = \rho_{\mathrm{DE}} + \rho_{\mathrm{DM}} \tag{12}$$

$$p_{\mathrm{DE}} = -2M_0 \,, \quad \rho_{\mathrm{DE}} = 2M_0 \,; \quad p_{\mathrm{DM}} = 0 \,, \quad \rho_{\mathrm{DM}} = \rho_0 \,, \tag{13}$$

i.e., the dark matter component is a dust fluid flowing along geodesics. This is explicit unification of dark energy and dark matter originating from the dynamics of a single scalar field - the "darkon" u.

3 Quintessential Inflation via Two Non-Riemannian Volume-Forms

Let us now consider, following [9], a modified-measure gravity-matter theory constructed in terms of two different non-Riemannian volume-forms (using again units where $G_{\mathrm{Newton}} = 1/16\pi$):

$$S = \int d^4x\, \Phi(A) \Bigl[R + L_1(\varphi, X)\Bigr] + \int d^4x\, \Phi(B) \Bigl[L_2(\varphi, X) + \frac{\Phi(H)}{\sqrt{-g}}\Bigr] \,. \tag{14}$$

Here the following notations are used:

- $\Phi(A)$ and $\Phi(B)$ are two independent non-Riemannian volume-forms:

$$\Phi(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \,, \tag{15}$$

- $\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}$ is the dual field-strength of an additional auxiliary tensor gauge field $H_{\nu\kappa\lambda}$ crucial for the consistency of (14).

- We are using Palatini formalism: $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$, where $g_{\mu\nu}$ and the affine connection $\Gamma^{\lambda}_{\mu\nu}$ are *apriori* independent.
- $L_{1,2}(\varphi, X)$ denote two different Lagrangians of a single scalar matter field φ - the "inflaton", of the form:

$$\begin{aligned} L_1(\varphi,X) &= X - V_1(\varphi)\,, \\ &\qquad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\,, \quad V_1(\varphi) = f_1\exp\{-\alpha\varphi\}\,, \\ L_2(\varphi,X) &= be^{-\alpha\varphi}X + U(\varphi)\,, \quad U(\varphi) = f_2\exp\{-2\alpha\varphi\}\,, \end{aligned} \tag{16}$$

  where α, f_1, f_2 are dimensionful positive parameters, whereas b is a dimensionless one.
- The form of the action (14) is fixed by the requirement of invariance under global Weyl-scale transformations

$$\begin{aligned} &g_{\mu\nu} \to \lambda g_{\mu\nu}\,, \quad \Gamma^{\mu}_{\nu\lambda} \to \Gamma^{\mu}_{\nu\lambda}\,, \quad \varphi \to \varphi + \frac{1}{\alpha}\ln\lambda\,, \\ &A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa}\,, \quad B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa}\,, \quad H_{\mu\nu\kappa} \to H_{\mu\nu\kappa}\,. \end{aligned} \tag{17}$$

Equations of motion w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$ yield a solution for the latter as a Levi-Civita connection

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_\nu\bar{g}_{\lambda\kappa} + \partial_\lambda\bar{g}_{\nu\kappa} - \partial_\kappa\bar{g}_{\nu\lambda}\right)\,, \tag{18}$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$

$$\bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu}\,, \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}}\,. \tag{19}$$

The metric $\bar{g}_{\mu\nu}$ plays an important role as the "Einstein frame" metric (see (22) below).

Variation of the action (14) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations

$$\partial_\mu\Big[R + L^{(1)}\Big] = 0\,, \quad \partial_\mu\Big[L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}}\Big] = 0\,, \quad \partial_\mu\Big(\frac{\Phi_2(B)}{\sqrt{-g}}\Big) = 0\,, \tag{20}$$

whose solutions read

$$\begin{aligned} &\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const}\,, \\ &R + L^{(1)} = M_1 = \text{const}\,, \\ &L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const}\,. \end{aligned} \tag{21}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants.

The first integration constant χ_2 in (21) preserves global Weyl-scale invariance (17) whereas the appearance of the second and third integration constants $M_1,\, M_2$ signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance under (17) due to the scale non-invariant solutions (second and third ones) in (21).

It is very instructive to elucidate the physical meaning of the three arbitrary integration constants $M_1,\, M_2,\, \chi_2$ from the point of view of the canonical Hamiltonian formalism (for details, we refer to [11]): $M_1,\, M_2,\, \chi_2$ are identified as conserved Dirac-constrained canonical momenta conjugated to (certain components of) the auxiliary maximal rank antisymmetric tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$ entering the original non-Riemannian volume-form action (14).

Performing transition from the original metric $g_{\mu\nu}$ to $\bar{g}_{\mu\nu}$ we arrive at the *"Einstein-frame"*, where the gravity equations of motion are written in the standard form of Einstein's equations

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu} \tag{22}$$

with an appropriate *effective* energy-momentum tensor given in terms of an Einstein-frame scalar Lagrangian $L_{\rm eff}$. The latter turns out to be of the non-canonical "k-essence" (kinetic quintessence) type [13] (containing higher powers of the scalar kinetic term $\bar{X}$)

$$L_{\rm eff} = A(\varphi)\bar{X} + B(\varphi)\bar{X}^2 - U_{\rm eff}(\varphi)\,, \quad \bar{X} \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\,, \tag{23}$$

where (recall $V_1 = f_1 e^{-\alpha\varphi}$ and $U = f_2 e^{-2\alpha\varphi}$)

$$A(\varphi) \equiv 1 + \frac{1}{2}be^{-\alpha\varphi}\frac{V_1(\varphi)+M_1}{U(\varphi)+M_2}\,, \quad B(\varphi) \equiv -\frac{\chi_2 b^2 e^{-2\alpha\varphi}}{4\big(U(\varphi)+M_2\big)}\,, \tag{24}$$

$$U_{\rm eff}(\varphi) \equiv \frac{(V_1(\varphi)+M_1)^2}{4\chi_2\big(U(\varphi)+M_2\big)}\,. \tag{25}$$

As a most remarkable feature, the effective scalar potential $U_{\rm eff}(\varphi)$ (25) possesses two *infinitely large flat regions*

- *(-) flat region* – for large negative values of φ, describing the "early" (inflationary) Universe:

$$U_{\rm eff}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2}{4\chi_2\, f_2}\,, \tag{26}$$

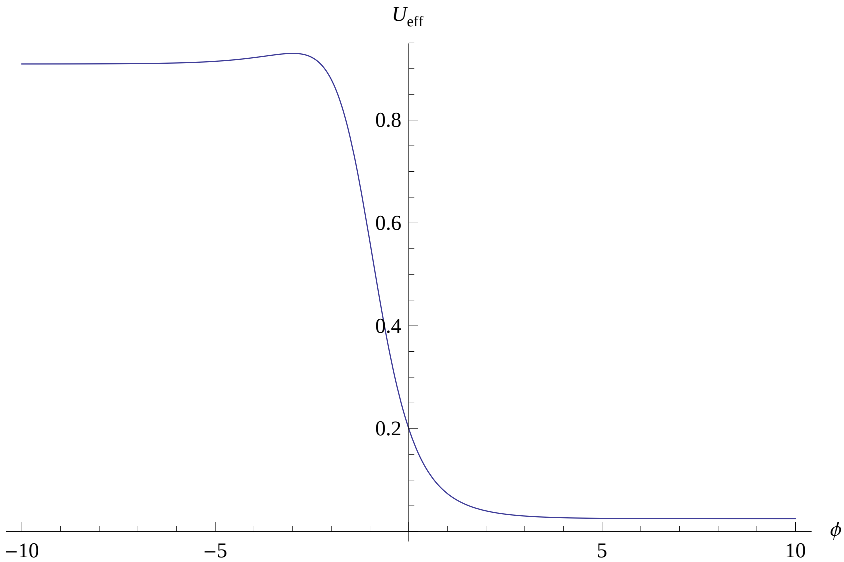

Figure 1. Qualitative shape of the effective scalar potential $U_{\rm eff}$ (25) as function of φ.

- *(+) flat region* – for large positive values of φ, describing the "late" (nowadays) Universe

$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2}{4\chi_2\, M_2} \,, \tag{27}$$

From the expression for $U_{\rm eff}(\varphi)$ (25) and Figure 1 we deduce that we have an *explicit realization of quintessential inflation scenario* [10] – continuously connecting an inflationary phase in the "early" Universe to a slowly accelerating expansion of "present-day" Universe [14] through the evolution of a single scalar field.

The flat regions (26) and (27) correspond indeed to the evolution of the "*early*" and the "*late*" Universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2} \,, \tag{28}$$

which makes the *vacuum energy density of the early Universe* $U_{(-)}$ *much bigger than that of the late Universe* $U_{(+)}$ (cf. (26), (27)).

If we choose the scales $M_1 \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$, where M_{EW}, M_{Pl} are the electroweak and Plank scales, respectively, we are then naturally led to a very small vacuum energy density

$$U_{(+)} \sim M_{EW}^8/M_{Pl}^4 \sim 10^{-120} M_{Pl}^4 \,, \tag{29}$$

which is the right order of magnitude for the present epoch's vacuum energy density as already realized in [15].

On the other hand, if we take the order of magnitude of the coupling constants in the effective potential $f_1 \sim f_2 \sim (10^{-2} M_{Pl})^4$, then the order of magnitude of the vacuum energy density of the early Universe becomes

$$U_{(-)} \sim f_1^2/f_2 \sim 10^{-8} M_{Pl}^4 \,, \tag{30}$$

which conforms to the Planck Collaboration data [16] implying the energy scale of inflation of order $10^{-2} M_{Pl}$.

4 Quintessential Inflation and Unified Dark Energy and Dark Matter

Now we will extend our results from the previous two sections by considering a combination of the both models above (14) and (1) – gravity coupled to both "inflaton" and "darkon" scalar fields within the non-Riemannian volume-form formalism, as well as we will also add coupling to the bosonic sector of the electro-weak model

$$S = \int d^4x\, \Phi(A)\Big[g^{\mu\nu} R_{\mu\nu}(\Gamma) + L_1(\varphi, X) - g^{\mu\nu}\big(\nabla_\mu \sigma_a\big)^* \nabla_\nu \sigma_a - V_0(\sigma)\Big] \\ + \int d^4x\, \Phi(B)\Big[L_2(\varphi, X) - \frac{1}{4g^2} F^2(\mathcal{A}) - \frac{1}{4g'^2} F^2(\mathcal{B}) + \frac{\Phi(H)}{\sqrt{-g}}\Big] \\ + \int d^4x \big(\sqrt{-g} + \Phi(C)\big) L(u, Y) \,. \tag{31}$$

Here we are using the same notations as in (15)-(16), (2)-(3) and in addition

- $\sigma \equiv (\sigma_a)$ is a complex $SU(2) \times U(1)$ iso-doublet scalar field with the isospinor index $a = +, 0$ indicating the corresponding $U(1)$ charge. The gauge-covariant derivative acting on σ reads

$$\nabla_\mu \sigma = \Big(\partial_\mu - \frac{i}{2}\tau_A \mathcal{A}_\mu^A - \frac{i}{2}\mathcal{B}_\mu\Big)\sigma \,, \tag{32}$$

  with $\frac{1}{2}\tau_A$ (τ_A – Pauli matrices, $A = 1, 2, 3$) indicating the $SU(2)$ generators and $\mathcal{A}_\mu^A$ ($A = 1, 2, 3$) and $\mathcal{B}_\mu$ denoting the corresponding $SU(2)$ and $U(1)$ gauge fields.

- The "bare" σ-field potential is of the same form as the standard Higgs potential

$$V_0(\sigma) = \frac{\lambda}{4}\left((\sigma_a)^* \sigma_a - \mu^2\right)^2 \,. \tag{33}$$

- The gauge field kinetic terms are (all indices $A, B, C = (1,2,3)$)

$$
\begin{aligned}
F^2(\mathcal{A}) &\equiv F^A_{\mu\nu}(\mathcal{A}) F^A_{\kappa\lambda}(\mathcal{A}) g^{\mu\kappa} g^{\nu\lambda} , \\
&\quad F^A_{\mu\nu}(\mathcal{A}) = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu + \epsilon^{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu , \\
F^2(\mathcal{B}) &\equiv F_{\mu\nu}(\mathcal{B}) F_{\kappa\lambda}(\mathcal{B}) g^{\mu\kappa} g^{\nu\lambda} , \\
&\quad F_{\mu\nu}(\mathcal{B}) = \partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu .
\end{aligned}
\tag{34}
$$

Following the same steps as above, we derive from (31) the physical *Einstein-frame* theory w.r.t. Weyl-rescaled Einstein-frame metric $\bar{g}_{\mu\nu}$ (19) and perform an additional "darkon" field redefinition $u \to \widetilde{u}$

$$
\frac{\partial \widetilde{u}}{\partial u} = \big(V(u) - 2M_0\big)^{-\frac{1}{2}} ; \quad Y \to \widetilde{Y} = -\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \widetilde{u} \partial_\nu \widetilde{u} . \tag{35}
$$

The Einstein-frame action reads

$$
S = \int d^4x \sqrt{-\bar{g}} \Big[R(\bar{g}) + L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}; \sigma, \mathcal{A}, \mathcal{B}\big) \Big] , \tag{36}
$$

where (recall $\bar{X} = -\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$)

$$
\begin{aligned}
L_{\rm eff}\big(\varphi, \bar{X}, \widetilde{Y}; \sigma, \mathcal{A}, \mathcal{B}\big) = \bar{X} - \widetilde{Y} \Big(V_1(\varphi) + V_0(\sigma) + M_1 - \chi_2 b e^{-\alpha\varphi} \bar{X} \Big) \\
+ \widetilde{Y}^2 \Big[\chi_2 (U(\varphi) + M_2) - 2M_0 \Big] + L[\sigma, \mathcal{A}, \mathcal{B}] ,
\end{aligned}
\tag{37}
$$

with

$$
L[\sigma, \mathcal{A}, \mathcal{B}] \equiv -\bar{g}^{\mu\nu} \big(\nabla_\mu \sigma_a\big)^* \nabla_\nu \sigma_a - \frac{\chi_2}{4g^2} \bar{F}^2(\mathcal{A}) - \frac{\chi_2}{4{g'}^2} \bar{F}^2(\mathcal{B}) . \tag{38}
$$

Tha Lagrangian (37) is again of a generalized "k-essence" form (non-linear w.r.t. both "inflaton" and "darkon" kinetic terms $\bar{X}$ and $\widetilde{Y}$). M_0 and M_1, M_2, χ_2 are the same integration constants as in (4) and (21), respectively.

The action (36)-(37) possesses an obvious Noether symmetry under the shift $\widetilde{u} \to \widetilde{u} + \text{const}$ with current conservation:

$$
\partial_\mu \Big(\sqrt{-\bar{g}} \bar{g}^{\mu\nu} \partial_\nu \widetilde{u} \frac{\partial L_{\rm eff}}{\partial \widetilde{Y}} \Big) = 0 , \tag{39}
$$

which is Einstein-frame counterpart of the original $g_{\mu\nu}$-frame "dust" dark matter density conservation (7).

For static (spacetime idependent) scalar field configurations (here the original "darkon" field u is static, whereas the transformed one $\widetilde{u}$ (35) is *not* – this is due to the dynamical Lagrangian "darkon" constraint (4)) we have

$$
\widetilde{Y} \big|_{\rm static} = \frac{V_1(\varphi) + V_0(\sigma) + M_1}{2\chi_2 \big(U(\varphi) + M_2\big) - 4M_0} , \tag{40}
$$

which upon substitution into (37) yields the following total scalar field effective potential (cf. Eq.(25)):

$$U_{\rm eff}\big(\varphi,\sigma\big) = \frac{\Big(V_1(\varphi) + V_0(\sigma) + M_1\Big)^2}{4\big[\chi_2(U(\varphi) + M_2) - 2M_0\big]}\,. \tag{41}$$

As for the purely "inflaton" potential (25), the "inflaton+Higgs" potential (41) similarly possess two infinitely large regions: $(-)$ flat region for large negative and $(+)$ flat region and large positive values of the "inflaton", respectively, as in Fig.1 (when σ is fixed).

- In the $(+)$ flat region (41) reduces to (cf. (27))

$$U_{\rm eff}\big(\varphi,\sigma\big) \simeq U_{(+)}(\sigma) = \frac{\Big(\frac{\lambda}{4}\left((\sigma_a)^*\sigma_a - \mu^2\right)^2 + M_1\Big)^2}{4\big(\chi_2 M_2 - 2M_0\big)}\,, \tag{42}$$

  which obviously yields as a lowest lying vacuum the Higgs one

$$|\sigma| = \mu\,, \tag{43}$$

  i.e., in the "late" (post-inflationary) Universe we have the standard spontaneous breakdown of $SU(2)\times U(1)$ gauge symmetry. Moreover, at the Higgs vacuum (43) we obtain from (42) a dynamically generated cosmological constant $\Lambda_{(+)}$ of the "late" Universe

$$U_{(+)}(\mu) \equiv 2\Lambda_{(+)} = \frac{M_1^2}{4\big(\chi_2 M_2 - 2M_0\big)}\,. \tag{44}$$

- In the $(-)$ flat region (41) reduces to the same expression as in (26), which is σ-field independent. Thus, the Higgs-like iso-doublet scalar field σ_a remains *massless* in the "early" (inflationary) Universe and accordingly there is *no* electro-weak spontaneous symmetry breaking there.

To study cosmological implications of (31) we perform a Friedmann-Lemaitre-Robertson-Walker (FLRW) reduction to the class of FLRW metrics

$$ds^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + a^2(t)d\boldsymbol{x}.d\boldsymbol{x} \tag{45}$$

and take the "inflaton" and "darkon" to be time-dependent only

$$\bar{X} = \frac{1}{2}\,\dot{\varphi}^2 \quad , \quad \widetilde{Y} = \frac{1}{2}v^2\,, \quad v \equiv \frac{d\widetilde{u}}{dt}\,. \tag{46}$$

Upon variation w.r.t. "lapse" $N(t)$ we take the usual gauge $N(t)=1$.

Now, the FLRW reduction of the "darkon" $\widetilde{u}$-eqs. of motion (39) yields a *cubic algebraic* eq. for its velocity v

$$\Big[\chi_2(U(\varphi)+M_2)-2M_0\Big]v^3 \\ -v\Big(V_1(\varphi)+V_0(\sigma)+M_1-\chi_2 b e^{-\alpha\varphi}\frac{1}{2}\,\dot{\varphi}^2\Big)-\frac{c_0}{a^3}=0\,, \tag{47}$$

where c_0 is an integration constant – the conserved Noether charge of (39) ("dust" dark matter particle number).

The equations of motion w.r.t. $N(t)$ and $a(t)$ (1st and 2nd Friedmann eqs.) read

$$\frac{\dot{a}^2}{a^2}=\frac{1}{6}\,\rho\,,\quad \frac{\ddot{a}}{a}=-\frac{1}{12}\big(\rho+3p\big)\,, \tag{48}$$

where the energy density ρ and pressure p are given by

$$\rho=\frac{1}{2}\,\dot{\varphi}^2\,\Big(1+\frac{3}{4}\chi_2 b e^{-\alpha\varphi}v^2\Big)+\frac{v^2}{4}\big(V_1(\varphi)+V_0(\sigma)+M_1\big)+\frac{3}{4}\,\frac{c_0}{a^3}\,v\,, \tag{49}$$

$$p=\frac{1}{2}\,\dot{\varphi}^2\,\Big(1+\frac{1}{4}\chi_2 b e^{-\alpha\varphi}v^2\Big)-\frac{v^2}{4}\big(V_1(\varphi)+V_0(\sigma)+M_1\big)+\frac{1}{4}\,\frac{c_0}{a^3}\,v\,. \tag{50}$$

Finally, the equation of motion w.r.t. "inflaton" φ reads

$$0=\frac{d}{dt}\Big[a^3\,\dot{\varphi}\,\Big(1+\frac{\chi_2}{2}be^{-\alpha\varphi}v^2\Big)\Big]+\alpha\frac{\chi_2\,U(\varphi)\,c_0\,v}{\chi_2\big(U(\varphi)+M_2\big)-2M_0} \\ +\alpha a^3 v^2\Big\{\frac{\dot{\varphi}^2}{4}\chi_2 b e^{-\alpha\varphi}-\frac{1}{2}\big(V_1(\varphi)+V_1(\sigma)\big) \\ +\frac{\chi_2 U(\varphi)\,\Big[V_1(\varphi)+V_0(\sigma)+M_1-\chi_2 b e^{-\alpha\varphi}\,\dot{\varphi}^2/2\Big]}{2\,\big[\chi_2\big(U(\varphi)+M_2\big)-2M_0\big]}\Big\}. \tag{51}$$

First, let us consider the $(+)$ flat region (27) of the inflaton potential (41) (right flat region on Fig.1) for large positive values of φ corresponding to the "late" (nowadays) Universe. In this case we have from (47), (49) and (50) (taking into account (33) and (43))

$$v=\Big[\frac{M_1}{\chi_2 M_2-2M_0}\Big]^{\frac{1}{2}}+\frac{1}{2M_1}\,\frac{c_0}{a^3}+\mathrm{O}\big(\frac{c_0^2}{a^6}\big)\,, \tag{52}$$

$$\rho=\frac{M_1^2}{4(\chi_2 M_2-2M_0)}+\frac{c_0}{a^3}\,\Big[\frac{M_1}{\chi_2 M_2-2M_0}\Big]^{\frac{1}{2}}+\mathrm{O}\big(\frac{c_0^2}{a^6}\big)\,, \tag{53}$$

$$p=-\frac{M_1^2}{4(\chi_2 M_2-2M_0)}+\mathrm{O}\big(\frac{c_0^2}{a^6}\big)\,. \tag{54}$$

Substituting (53) into the first Friedmann Eq.(48) we obtain (the solution for $a(t)$ below first appeared in [17])

$$a(t) \simeq \Big(\frac{\widetilde{C}_0}{2\Lambda_{(+)}}\Big)^{1/3} \sinh^{2/3}\Big(\sqrt{\frac{3}{4}\Lambda_{(+)}}\, t\Big)\,, \qquad \dot{\varphi} \simeq \text{const}\, \sinh^{-2}\Big(\sqrt{\frac{3}{4}\Lambda_{(+)}}\, t\Big)\,, \tag{55}$$

with $\Lambda_{(+)}$ as in (44) and $\widetilde{C}_0 \equiv c_0 \sqrt{M_1 (\chi_2 M_2 - 2M_0)^{-1}}$.

Relations (53) and (54) straightforwardly show that in the "late" (nowadays) Universe we have explicit unification of dark energy (given by the dynamically generated cosmological constant (44) – first terms on the r.h.s. of (53) and (54)), and dark matter given as a "dust" fluid contribution – second term on the r.h.s. of (53).

Next consider the $(-)$ flat region (26) of the inflaton potential (41) (left flat region on Fig.1) for large negative values of φ corresponding to the "early" ("inflationary") Universe. We will consider the "slow-roll" inflationary epoch [18] (*i.e.*, $\ddot{\varphi}, \dot{\varphi}^2, \dot{\varphi}^3, \ldots$ – ignored)

$$v = e^{\frac{1}{2}\alpha\varphi}\Big[v_1 + \frac{1}{2f_1}\frac{c_0}{a^3}\, e^{\frac{1}{2}\alpha\varphi} + \mathrm{O}\big(e^{\alpha\varphi}\big)\Big]\,, \quad v_1 \equiv -\Big(\frac{f_1}{\chi_2 f_2}\Big)^{\frac{1}{2}}\,, \tag{56}$$

$$\rho = U_{(-)} - e^{\frac{1}{2}\alpha\varphi}|v_1|\,\frac{c_0}{a^3} + \frac{1}{4}\widetilde{M} v_1^2 e^{\alpha\varphi} + \mathrm{O}\Big(e^{3\alpha\varphi/2}, \frac{c_0^2}{a^6}\Big)\,, \tag{57}$$

$$U_{(-)} \equiv \frac{f_1^2}{4\chi_2 f_2}\,, \quad \widetilde{M} \equiv M_1 + V_0(\sigma = 0) = M_1 + \frac{\lambda}{4}\mu^4\,. \tag{58}$$

Friedmann (48) and inflaton (51) equations can be solved analytically in the "slow-roll" approximation for the special relation among parameters $1 + \frac{bf_1}{2f_2} = \frac{2}{3}\alpha^2$. In the latter case we have

$$\dot{\varphi} - \frac{|v_1|}{2\alpha H_0}\Big[\frac{c_0}{c_1^3}\, e^{-3H_0 t} - \frac{1}{4}\widetilde{M}|v_1|\Big]\, e^{\alpha\varphi} = 0 \quad , \quad H_0 \equiv \sqrt{\frac{1}{6}U_{(-)}}\,, \tag{59}$$

where c_1 is another integration constant. For the inflaton field and Friedmann scale factor we obtain

$$e^{-\alpha\varphi(t)} = c_2 + \frac{|v_1|}{2\alpha H_0}\Big(\frac{c_0}{3c_1^3 H_0}\, e^{-3H_0 t} + \frac{1}{4}\widetilde{M}|v_1|\, t\Big)\,, \tag{60}$$

$$a(t) = c_1 e^{H_0 t}\, e^{-\frac{1}{6}\alpha\varphi(t)}\,, \tag{61}$$

where c_2 is a third integration constant.

Eqs.(59)-(61) display the effect of the presence of "dusty" dark matter ($c_0 \neq 0$) on the "slow-roll" inflationary evolution (here we must have $\dot{\varphi} \geq 0$):

- $\dot{\varphi}\,(t) > 0$ for $t < t_* \equiv \frac{1}{3H_0}\ln\bigl(4c_0(\widetilde{M}|v_1|c_1^3)^{-1}\bigr)$, where $\dot{\varphi}\,(t_*) = 0$, *i.e.*, $\varphi(t)$ rolls forward untill $t = t_*$.
- According to (61) the prefactor $e^{-\frac{1}{6}\alpha\varphi(t)}$ of the inflationary time exponential $e^{H_0 t}$ drops down with $t \leq t_*$.

For $t > t_*$ the evolution described by the inflaton solution (60) *cannot anymore* be valid, since according to (59) the inflaton velocity is negative for $t > t_*$, *i.e.*, for $t > t_*$ the inflaton would start rolling backwards. This non-validity of (60) is due to the fact that for $t \sim t_*$ the inflaton value $\varphi(t)$ exits the $(-)$ flat region of the inflaton effective potential (41) (left flat region on Fig.1). The latter sets the following constraint on the integration constant c_2 in (60) for the latter to be valid

$$e^{-\alpha\varphi(t_*)} \equiv c_2 + \frac{\widetilde{M}}{f_1}\Bigl[1 + \ln\Bigl(\frac{4c_0}{\widetilde{M}|v_1|c_1^3}\Bigr)\Bigr] < \frac{f_1(\chi_2 M_2 - 2M_0)}{f_2\chi_2\widetilde{M}} \equiv e^{-\alpha\varphi_{\max}}\;, \quad (62)$$

where $\varphi_{\max}$ is the location of the maximum of the inflaton potential (41) – the small "bump" on the left half of Fig.1, which is just outside the $(-)$ flat region.

Let us note that the relative height $\Delta U_{(-)}$ of the above mentioned "bump" of the inflaton potential (41) w.r.t. the height of the $(-)$ flat region (26):

$$\Delta U_{(-)} \equiv U_{\rm eff}(\varphi_{\max},\mu) - \frac{f_1^2}{4\chi_2 f_2} = \frac{\widetilde{M}^2}{4\bigl(\chi_2 M_2 - 2M_0\bigr)} \quad (63)$$

($\widetilde{M}$ as in (58)) is of the same order of magnitude as the small effective cosmological constant (44) in the $(+)$ flat region ("late" Universe).

5 Conclusions

The non-Riemannian volume-form formalism (*i.e.*, employing alternative non-Riemannian reparametrization covariant integration measure densities on the spacetime manifold) has substantial impact in any general-coordinate or reparametrization invariant field theories.

- The non-Riemannian volume-form formalism in gravity/matter theories naturally provides a self-consistent unified description of dark energy as dynamically generated cosmological constant and dark matter as a "dust" fluid flowing along geodesics realized through the dynamics of a single "darkon" scalar field. This unification becomes manifest within the "late" (dark energy dominated) epoch of the Universe's evolution.

- Employing two different non-Riemannian volume-forms leads to the construction of a new class of "quintessential" gravity-matter models, producing an effective scalar "inflaton" potential with two infinitely large flat regions. This allows for a unified description of both early Universe inflation as well as of present dark energy dominated epoch.
- The above non-conventional "quintessential" gravity-matter models can be extended to include both the "darkon" as well as the fields comprising the bosonic sector of the electroweak theory, in particular – a Higgs-like scalar σ, whereby producing *dynamically* in the post-inflationary epoch an effective potential for σ of the canonical electroweak symmetry breaking Higgs form, while keeping the electroweak gauge symmetry intact in the early inflationary Universe.

Let us also note that application of the non-Riemannian volume-form formalism in the context of minimal $N = 1$ supergravity [19] naturally generates a *dynamical cosmological constant* as an arbitrary dimensionful integration constant, which triggers *spontaneous supersymmetry breaking* and mass generation for the gravitino – a new mechanism for the *supersymmetric Brout-Englert-Higgs effect*.

Acknowledgements

We gratefully acknowledge support of our collaboration through the academic exchange agreement between the Ben-Gurion University and the Bulgarian Academy of Sciences. S.P. and E.N. have received partial support from European COST actions MP-1210 and MP-1405, respectively, as well from Bulgarian National Science Fund Grant DFNI-T02/6. E.G. received partial support from COST Action CA-15117.

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